We have the nonlinear state space representation,

where,

and , where our input

In other words, we have,

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where all these functions are nonlinear functions of time. Remembering the multivariate version of the Taylor Series Expansion, for example for a simple two variable case, linearizing about and for the first order Taylor Polynomial,

We want to linearize about the equilibrium point,

, and

For our situation, taking a multivariate first order Taylor Series approximation (but with six variables, not just two as shown above), we get:

This approximation holds well only when the difference between the state and equilibrium point is small, for example, is small, because we have discarded all Higher Order Terms (H.O.T.). The HOT containing, for example say, , or , are all very small if is very small, hence the approximation that the HOT are zero holds well only if this is the case. Note the symbol above simply denotes multiplication, and the “bar” symbol denotes each partial should be evaluated at the equilibrium state. We can write this more compactly using matrices, (note we moved the constant term in the front of the equation further back):

In the above equations we already know that for example equals the derivative of our state evaluated at the equilibrium point, or . Making this substitution, we get:

Well, we know that by definition the equilibrium point should not change once it’s reached there, in other words, the rate of change at the equilibrium point should be zero. Therefore, the entire term goes to zero.

Next, in our specific case, all the states at the equilibrium point we have selected are zero, as shown in our equation for earlier. Making this substitution:

Finally, we can call these matrices, which are the Jacobian evaluated at our equilibrium point, the A and B matrices. These are constant matrices because we evaluate them at the equilibrium point. In general, linearizing about a specific equilibrium point results in constant A and B matrices and hence a Linear Time Invariant (LTI) system, whereas linearizing about a trajectory results in a Linear Time Varying (LTV) system.

We have linearized the nonlinear state space representation to obtain the linear state space representation. As mentioned earlier this linear approximation holds for small differences between the state and the equilibrium point.